

Engineering Notes

Extended Nonlinear Lifting-Line Method for Aerodynamic Modeling of Reconfigurable Aircraft

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DOI: 10.2514/1.C031406

Nomenclature

C_l	=	section lift coefficient
c	=	local chord length
D	=	fading-memory constant
G	=	nondimensional circulation
l	=	section lift force/length
m	=	number of points used in sine series expansion of circulation function
N	=	consecutive iterations convergence parameter
r	=	perpendicular distance from vortex
tol	=	iteration relative tolerance of conversion
U_∞	=	freestream velocity magnitude
w	=	downwash velocity
y_0	=	wing semispan, y coordinate of wingtip
α	=	wind incidence angle, wing angle of attack
α_{eff}	=	effective angle of attack
α_{2D}	=	2-D vortex-induced wind incidence angle
α_{3D}	=	3-D wake-induced wind incidence angle
Γ	=	circulation magnitude
η	=	nondimensional spanwise coordinate
θ	=	local wing twist angle
Λ	=	local wing sweep angle
ξ	=	nondimensional chordwise coordinate
ρ	=	air density
ϕ	=	trigonometric spanwise coordinate

I. Introduction

THE preliminary design of morphing aircraft [1–7] requires aerodynamic analyses at widely varying flight conditions and geometric configurations. Thus, a fast, adaptable algorithm is required that can accept geometric “morphing” parameter variations and recompute the wing (or aircraft) aerodynamic characteristics. Lifting-line theory, first developed by Prandtl and Tietjins [8] and Munk [9], is chosen because it fulfills these needs and is readily programmed to populate large, multidimensional lookup tables for reconfigurable aircraft simulation [7]. The basis of the lifting-line technique used in the present study is Weissinger’s method for straight, swept wings [10]. This method uses a flow-tangency (i.e.,

impermeability) condition at the three-quarter-chord curve of the wing, which avoids the singularity Prandtl’s method introduces for swept wings when the effects of bound vorticity are incorporated.

Prandtl’s and, indeed, Weissinger’s methods assume that the airfoil section lift curves are linear (that is, the lift coefficient at any angle of attack is fully described by the zero-lift angle of attack and the lift-curve slope) and that the section drag coefficients are identically zero, i.e., that the flow is inviscid. A common method to incorporate nonlinear sectional data is to iterate on the lifting-line formulation via a weighted averaging technique that blends the linearized potential flow over the wing with the nonlinear sectional data. This iterative procedure originates with the work of Tani [11] and Multhopp [12], who have developed a numerical integration technique similar to Gaussian quadrature for computing the net lift contribution of the spanwise circulation distribution using sine series. The coefficients of these series have been formalized and tabulated by Sivells and Neely [13]. More recently, this method has been used to calculate the aerodynamic characteristics of drooped leading-edge wings around stall [14]. It should be noted that the above studies have been restricted to straight wings so that Prandtl’s formulation can be used. Owens [15] has applied this iterative technique to Weissinger’s method for straight, swept wings.

Several extensions/generalizations to Weissinger’s method have been developed subsequently. Wing quarter-chord curves described by polynomial equations have been analyzed using modified lifting-line theory for stationary [16] and oscillating wings [17]. Wickenheiser and Garcia [18] have generalized this formulation to wings of arbitrary, piecewise continuous curvature. Phillips and Snyder [19] have developed a technique using 3-D vortex elements that uses Newton’s method to solve the resulting system of nonlinear equations.

The present study is based on modifications to the authors’ previous work on curved lifting lines [18]. First, a summary of the salient features of the generalized Weissinger’s method is given, followed by a presentation of the modifications to this method necessary to incorporate nonlinear section data. An iterative procedure is developed whose purpose is to cause the convergence of the results of the Weissinger’s method and the results using the sectional aerodynamic data. Subsequently, the results of this technique are compared with wind-tunnel data for elliptical and crescent wings at a Reynolds number $Re = 1.7 \times 10^6$ [20] and tapered wings at $Re = 3.49 \times 10^6$ [13].

II. Method Formulation

The lifting-line technique used in this study consists of two phases. In the first phase, a modified Weissinger method for planar, curved wings is used to compute an initial guess of the circulation distribution along the span and to populate the matrix of coefficients used in the downwash calculations in the second phase. The first phase is based on the extended lifting-line technique derived in [18] and uses linearized sectional data. The second phase consists of an iterative procedure that attempts to minimize the discrepancy between the circulation distributions predicted by 1) the lifting-line analysis and 2) the Kutta–Joukowski theorem using the nonlinear sectional data, which take into account viscous effects. Brief presentations of these two phases are given in the next sections.

A. Modified Weissinger Method (Initial Phase)

As previously mentioned, this phase is based on minor modifications to the lifting-line method derived and presented in detail in [18]; thus, only the key points and equations are given herein. The Cartesian coordinate system and the layout of a typical horseshoe

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